Journal of Hazardous Materials, 8 (1983) 121—128
Elsevier Science Publishers B.V., Amsterdam — Printed in The Netherlands

A SIMPLE, PHYSICAL CALCULATION OF THE RESPONSE OF A BURIED CONDUCTING DRUM TO ELECTROMAGNETIC INDUCTION SEARCHING

A.E. LORD, Jr.

Department of Physics and Atmospheric Sciences, Drexel University, Philadelphia, Pennsylvania 19104 (U.S.A.)

and R.M. KOERNER

Department of Civil Engineering, Drexel University, Philadelphia, Pennsylvania 19104 (U.S.A.)

(Received October 18, 1982; accepted in revised form January 5, 1983)

Summary

A very simple, physical calculation of the magnitude of the response of a single buried conducting drum to electromagnetic induction searching is given. The work is done for a frequency of 10 kHz, which means the skin depth in the soil is very much greater than the coil spacing of the search unit. The drum is modelled as a thin, conducting spherical shell, where the induced eddy currents act as a dipole. The results agree with actual experiments of electromagnetic searching for buried drums to better than an order of magnitude.

A number of geophysical techniques are being used to detect buried drums and seepage plumes, both related to the hazardous materials waste site problem [1,2,3]. A recent paper [4] has dealt in considerable detail with the use of an electromagnetic induction method for detecting buried steel drums of various sizes and depths of bury in a quite homogeneous sandy soil. In reference [4] the general response for a buried sphere with induction searching was reviewed from the literature. The general response is very complicated and physical insight suffers considerably due to the complexity. In the present note a simple, physical calculation is made of the response of a buried drum to electromagnetic induction searching.

The approximate shape (not magnitude) of the response of a buried conducting sphere to an induction unit with horizontal transmitter and receiver was derived in simple form in [4]. The basis of the calculation is shown on Fig. 1. The transmitter acting as a dipole (transmitter coil dimensions $\ll r_1$) induces eddy currents in the buried metal object. These eddy currents produce a magnetic dipole moment m_d which is proportional to $(2h^2 - x^2)/r_1^5$ (from simple dipole theory [8]). This dipole moment now in turn induces an emf (induced voltage) in the receiver coil which is proportional to $m_d\,[2h^2 - (D-x)^2]/r_2^5$. Thus the emf, ϕ, developed in the receiver as a function

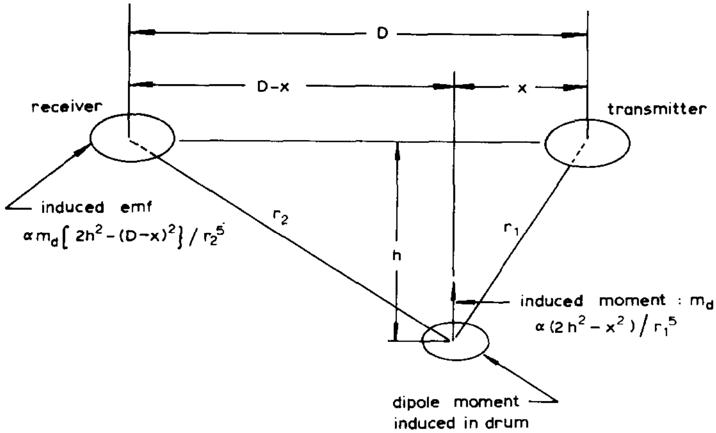

Fig. 1. Dipole model for calculating spatial dependence of buried drum response.

of x, is given by

$$\phi = C \frac{(2h^2 - x^2)(2h^2 - (D-x)^2)}{[(h^2 + x^2)(h^2 + (D-x)^2)]^{5/2}} \tag{1}$$

The constant C depends on the nature of the transmitter and receiver coils, the frequency, the actual details of the buried object, etc. Equation (1), although not exact, gives a reasonably good indication of the exact theoretical response encountered when the buried object is a simple shape (sphere, long cylinder, etc. [5]). It also is typical of the responses encountered in the field when inductively searching for isolated drums [4]. Figure 2 shows this spatial dependence (from eqn. (1)) for typical depths of dipole burial.

The exact responses for buried objects are quite complicated and are given in Keller and Frischknecht [5] and Grant and West [6]. McNeill [7] has presented the results of available theory for the responses of the vertical (V) and horizontal (H) coil configurations (Fig. 3) for a uniformly conducting half space (ref. [5], p. 335):

$$\left(\frac{H_s}{H_p}\right)_V = \frac{2}{(\gamma s)^2} \{9 - [9 + 9\gamma s + 4(\gamma s)^2 + (\gamma s)^3]\, e^{-\gamma s}\} \tag{2}$$

$$\left(\frac{H_s}{H_p}\right)_H = 2\left[1 - \frac{3}{(\gamma s)^2} + [3 + 3\gamma s + (\gamma s)^2]\, \frac{e^{-\gamma s}}{(\gamma s)^2}\right], \tag{3}$$

where H_s is the magnetic field detected in the receiver coil from the eddy currents induced in the earth; H_p is the magnetic field detected in the receiver coil directly from the transmitting coil; $\gamma = \sqrt{i\omega\mu_0\sigma}$; $\omega = 2\pi f$; f = frequency; σ = electrical conductivity; μ_0 = permeability of free space; $i = \sqrt{-1}$; and s = inter-coil spacing.

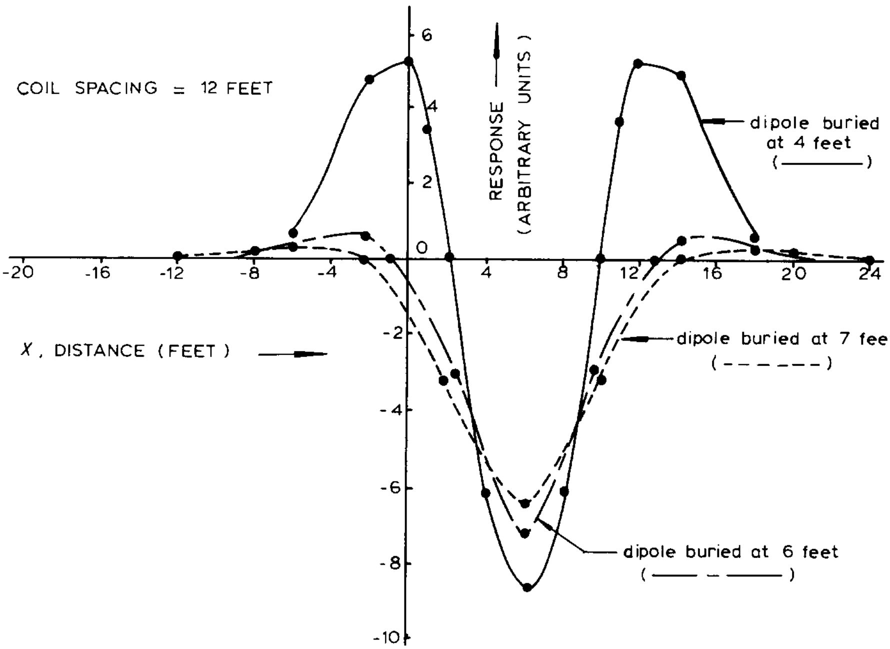

Fig. 2. Response of buried dipole as a function of position of coils.

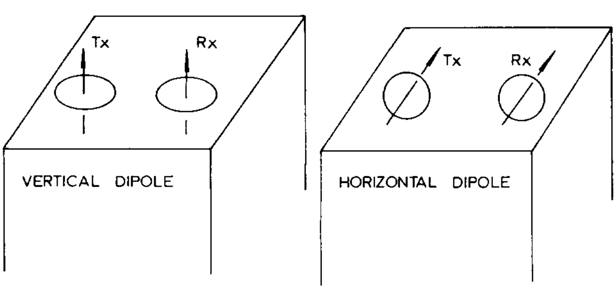

Fig. 3. Vertical and horizontal dipole configurations.

This analysis assumes that the separation between coils is much greater than the coil dimensions themselves [5].

The skin depth is given by

$$\delta = \sqrt{\frac{2}{\omega\mu_0\sigma}} = \frac{\sqrt{2i}}{\gamma} \tag{4}$$

Thus

$$\gamma s = \sqrt{2i}\, s/\delta\ . \tag{5}$$

The ratio of intercoil spacing to the skin depth is defined as the induction number, B,

$$\gamma s = \sqrt{2i}\,B\,. \tag{6}$$

If the induction number is very small ($\gamma s \ll 1$), i.e., the coil spacing is very much less than the skin depth, then eqns. (2) and (3) both reduce to [7]:

$$\left(\frac{H_s}{H_p}\right)_V = \left(\frac{H_s}{H_p}\right)_H = \tfrac{1}{2}\,iB^2 = \tfrac{1}{4}\,i\omega\mu_0\sigma s^2 \tag{7}$$

The magnitude of the secondary magnetic field is now directly proportional to the ground conductivity and the phase of secondary magnetic field leads the primary magnitude field by 90°. (This is the so-called quadrature component of the field.) Equation (7) is the basis of electromagnetic terrain conductivity measuring units. For a strongly conducting body, such as a metal, the main response should be the in-phase component.

In searching for a buried metal (conducting) drum, the conductivity reading on the unit is not the conductivity of the metal, but rather the effective (H_s/H_p) that the meter "sees" in encountering the drum. It is this (H_s/H_p) ratio that shall be calculated in this note. That is, a simple model of the drum will be used and the C in eqn. (1) will be estimated.

Consider a horizontal dipole transmitter (a circular coil) and a horizontal dipole receiver (Fig. 4). The drum is modelled (approximated) as a thin conducting spherical shell. The drum is located midway between the two coils; in this position the response is a maximum (Refer to Fig. 2). The horizontal dipole transmitter produces a radial component of magnetic field [8]

$$H_r = \frac{2m}{4\pi r^3}\cos\theta \tag{8}$$

and a tangential component [8]

$$H_\theta = \frac{m}{4\pi r^3}\sin\theta \tag{9}$$

Here $m = IA$ (Fig. 4). These components will produce a resultant field

$$|\vec{H}_T| = \frac{m}{4\pi r^3}\sqrt{3\cos^2\theta + 1} \tag{10}$$

at angle α to the radial direction (Fig. 4).

The angle is given by

$$\alpha = \tan^{-1}(H_\theta/H_r) \tag{11}$$

Eddy currents will move around the shell in circles whose normals are per-

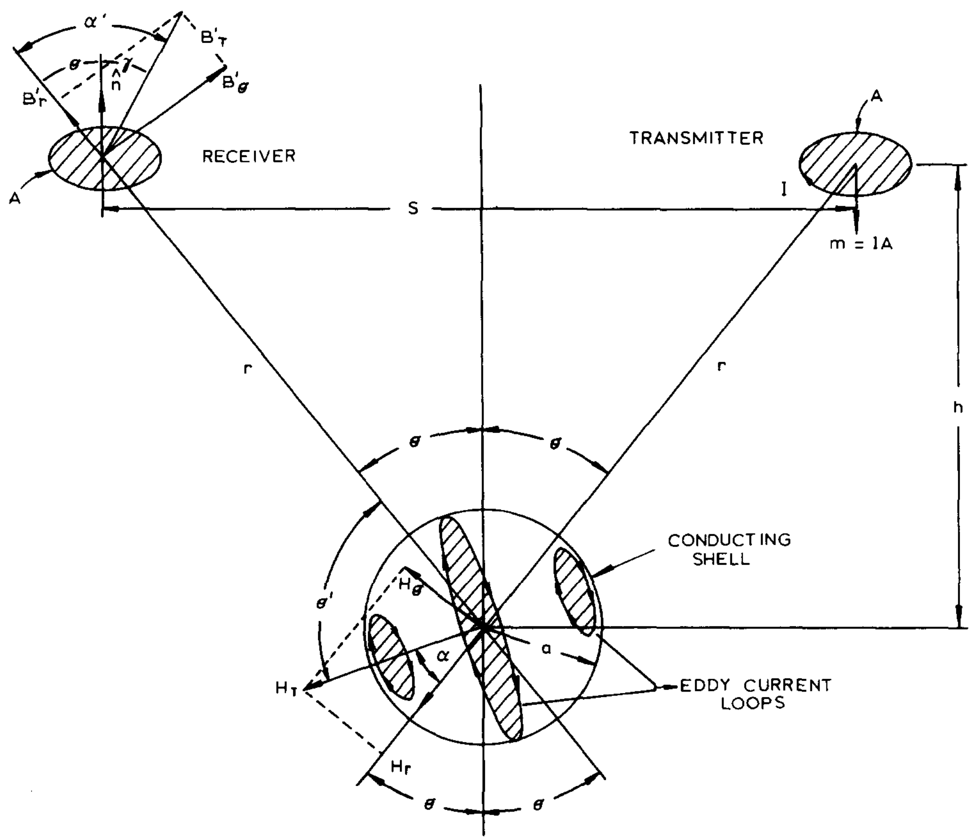

Fig. 4. Model of buried drum as a conducting, spherical shell, producing a dipole field.

pendicular to $\vec{H}_T$. If the skin depth in the steel drum is a very small fraction of the wall thickness (this is certainly not completely true, but probably a reasonable approximation)*, then the inside of the shell is completely shielded from the magnetic induction ($\vec{B}$), and the magnetic moment induced in the spherical shell is [6,9]

$$\vec{m}' = 2\pi a^3 \vec{H}_T \tag{12}$$

where a is the radius of the sphere. The sphere is now treated as a dipole and the components of the magnetic induction at the receiver are

$$B_r' = \left(\frac{\mu_0}{4\pi}\right) \frac{2m'}{r^3} \cos\theta' \tag{13}$$

*In using μ_0 instead of the actual ferromagnetic permeability μ, which is certainly much greater than μ_0, the skin depth has been significantly overestimated.

$$B'_\theta = \left(\frac{\mu_0}{4\pi}\right)\frac{m'}{r^3}\ \sin\theta' \tag{14}$$

where, from Fig. 4,

$$\theta' = 180° - 2\theta - \alpha \tag{15}$$

As before, the total induction at the receiver is of magnitude

$$|\vec{B}'_T| = \frac{\mu_0 m'}{4\pi r^3}\ \sqrt{3\cos^2\theta' + 1} \tag{16}$$

and, using eqns. (10) and (12),

$$|\vec{B}'_T| = \frac{\mu_0 a^3 m}{8\pi r^6}\ \sqrt{(3\cos^2\theta + 1)(3\cos^2\theta' + 1)} \tag{16a}$$

This total induction makes an angle

$$\alpha' = \tan^{-1}(B'_\theta / B'_r) \tag{17}$$

with the radial (Fig. 4). The induced response of the transmitter to this secondary field will be proportional to

$$(\vec{B}'_T)\cdot A\hat{n} \tag{19}$$

where A is the area of the receiving coil and $\hat{n}$ is the unit vector normal to the receiver coil area; the primary response will be proportional to

$$(\vec{B}_p)\cdot A\hat{n} \tag{18}$$

From formulas similar to eqns. (8) and (9)

$$|\vec{B}_p| = \left(\frac{\mu_0}{4\pi}\right)\frac{m}{s^3}\quad \text{(in this case, } \sin\theta = 1) \tag{20}$$

Hence the sought after ratio is

$$\left(\frac{H_s}{H_p}\right)_H = \frac{\vec{B}'_T\cdot A\hat{n}}{\vec{B}_p\cdot A\hat{n}} = \frac{\mu_0 a^3 m/8\pi r^6\sqrt{(3\cos^2\theta + 1)(3\cos^2\theta' + 1)}A\cos\gamma}{A\,\mu_0 m/4\pi s^3} \tag{21}$$

$$\left(\frac{H_s}{H_p}\right)_H = \frac{a^3 s^3}{2r^6}\ \sqrt{(3\cos^2\theta + 1)\,(3\cos^2\theta' + 1)}\ \cos\gamma \tag{21a}$$

where, from Fig. 4,

$$\gamma = \alpha' - \theta. \tag{22}$$

Combining eqns. (7) and (21), the following is obtained for the value of the apparent conductivity anomaly due to the drum as measured with an electromagnetic conductivity meter working at very low induction numbers:

$$\sigma_a = \frac{2a^3 s}{\omega \mu_0 r^6} \sqrt{(3\cos^2\theta + 1)(3\cos^2\theta' + 1)} \cos\gamma \,. \qquad (23)$$

As an example calculation, the following parameters are considered: $s = 3.7$ meters (typical of a commercial unit); $h = 3$ meters (unit held 1 m above ground; center of drum buried 2 m below surface); $a = 1/2$ meter (approximately for 55 gallon drum); $\omega = 10{,}000$ Hz (typical of a commercial unit); $\mu_0 = 4\pi \times 10^{-7}$ kg m s^{-2} A^{-2} (SI-units). The result is:

σ_a = 0.0062 mho/meter = 6.2 millimho/meter

The electromagnetic conductivity meter used in reference [4] reads in units of millimho/meter for the ground conductivity. The unit is usually used in the mode which measures the quadrature component ("out-of-phase" component). The metal drum should produce a response mainly "in-phase" with the primary field. Hence the results of reference [4], which dealt entirely with the quadrature component are not directly applicable to this calculation.

Measurements were subsequently made of the "in-phase" component over the drum buried with six feet of cover. The response was complete downward pinning of the meter directly over the drum and a positive lobe of height about 0.5 millimho/meter. Figure 2 gives the ratio of the negative dip to the positive lobe as about 11. Hence the depth of the negative dip would be 11 × 0.5 = 5.5 millimho/meter, which is very close to the 6.2 millimho/meter calculated above. This almost exact agreement should not be taken too seriously — the results differ by about a factor of two for the 3.5 feet drum burial — but do indicate that our simple model for the spatial dependence and magnitude of the drum response is close to physical reality.

Acknowledgements

Discussions with J.D. McNeill proved quite helpful. Thanks are due to the Oil and Hazardous Material Spills Branch, Municipal/Environmental Research Laboratory of the Environmental Protection Agency, Edison, NJ, for financial support. Special thanks are due to Dr. John Brugger for his constant encouragement and support.

References

1 A.E. Lord, Jr., S. Tyagi, R.M. Koerner, J.J. Bowders, J.E. Sankey and S. Cohen, Use of nondestructive testing methods to detect and locate buried containers verified by ground truth measurements, Proc. 1982 Hazard. Mater. Spills Conf., Milwaukee, WI, April 1982, Information Transfer, Inc., Silver Spring, MD, pp. 185—191.

2 A.E. Lord, Jr. and R.M. Koerner, Electromagnetic methods in subsurface investigations, Proc. Engr. Found. Conf. on Updating Subsurface Sampling of Soils and Rocks and Their In-Situ Testing, Santa Barbara, CA, January 1982 (to be published by The Engineering Foundation, New York)

3 R.C. Benson and R.A. Glaccum, Remote assessment of pollutants in soil and groundwater, Proc. 1979 Conf. on Hazard. Mater. Risk Assessment, Disposal and Management, Miami Beach, FL, April 1979, Information Transfer, Inc., Silver Spring, MD, pp. 188—194.

4 S. Tyagi, A.E. Lord, Jr. and R.M. Koerner, Use of a very-low-frequency electromagnetic method at 9.5 kHz to detect buried drums in sandy soil, J. Hazardous Materials, 7 (1983) 353–373.
5 G.V. Keller and F.C. Frischknecht, Electrical Methods in Geophysical Prospecting, Pergamon, New York, 1966.
6 F.S. Grant and G.F. West, Interpretation Theory in Applied Geophysics, McGraw-Hill, New York, 1965.
7 J.D. McNeill, Electromagnetic terrain conductivity measurements at low induction numbers, Technical Note TN-6, Geonics Ltd., Mississauga, Ontario, Canada, October 1980.
8 P. Lorrain and D.R. Corson, Electromagnetic Fields and Waves, W.H. Freeman Co., San Francisco, 1970.
9 G. Wouch and A.E. Lord, Jr., Eddy currents: levitation, metal detectors and induction heating, Am. J. Phys., 46 (1978) 464—466.